

# Fractional Gaussian Noise (FGN)

## Parametrization

The (stationary) FGN (Gaussian) process has correlation function at lag  $k$

$$\rho(k) = |k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}$$

where  $H$  is the Hurst parameter or self-similarity parameter, which we assume to be

$$1/2 \leq H < 1.$$

so the process has long range properties for  $H > 1/2$ . The locations of the the process is fixed to  $1, 2, \dots, n$ , where  $n$  is the dimension of the finite representation of the FGN process.

## Hyperparameters

The marginal precision,  $\tau$ , of the process is represented as

$$\tau = \exp(\theta_1)$$

The Hurst parameter  $H$  is represented as

$$H = \frac{1}{2} + \frac{1}{2} \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior is defined on  $\theta = (\theta_1, \theta_2)$ .

## Specification

The FGN model is specified as

```
f(<whatever>, model="fgn", order=<order>, hyper = <hyper>)
```

The parmeter `order` gives the order of the Markov approximation. Currently, only `order=3` is implemented.

## Hyperparameter spesification and default values for model="fgn"

```
doc Fractional Gaussian noise model
```

```
hyper
```

```
  theta1
```

```
    hyperid 13101
```

```
    name log precision
```

```
    short.name prec
```

```
    prior pc.prec
```

```
    param 3 0.01
```

```
    initial 1
```

```
    fixed FALSE
```

```
    to.theta function(x) log(x)
```

```
    from.theta function(x) exp(x)
```

```
  theta2
```

```

    hyperid 13102
    name logit H
    short.name H
    prior pcfnh
    param 0.9 0.1
    initial 2
    fixed FALSE
    to.theta function(x) log((2 * x - 1) / (2 * (1 - x)))
    from.theta function(x) 0.5 + 0.5 * exp(x) / (1 + exp(x))

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 5

aug.constr 1

n.div.by

n.required FALSE

set.default.values TRUE

order.default 4

order.defined 3 4

pdf fgn

```

Hyperparameter spesification and default values for model="fgn2"

doc Fractional Gaussian noise model (alt 2)

hyper

theta1

```

    hyperid 13111
    name log precision
    short.name prec
    prior pc.prec
    param 3 0.01
    initial 1
    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

```

theta2

```

    hyperid 13112
    name logit H
    short.name H

```

```

prior pcfgnh
param 0.9 0.1
initial 2
fixed FALSE
to.theta function(x) log((2 * x - 1) / (2 * (1 - x)))
from.theta function(x) 0.5 + 0.5 * exp(x) / (1 + exp(x))

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 4

aug.constr 1

n.div.by

n.required FALSE

set.default.values TRUE

order.default 4

order.defined 3 4

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```

## Example

```

library(longmemo)
n = 1000
H = 0.77
y = simFGNO(n, H)
y = y - mean(y)
r = inla(y ~ -1 + f(idx, model="fgn"),
        data = data.frame(y, idx=1:n),
        control.family = list(hyper = list(prec = list(initial = 12, fixed=TRUE))))
print(c(Post.mean=r$summary.hyperpar[2,"mean"],
        Post.mode=r$summary.hyperpar[2,"mode"]))

```

## Notes

In the example above, then the `f(idx,model="fgn")` object will expand into a Gaussian of length  $(\text{order} + 1) * n$ . The first  $n$  elements is the FGN model (which is of interest), then there are `order` vector of AR1 processes each of length  $n$ , and the sum of these AR1 processes is used to represent the FGN.

Another alternative, is `f(idx,model="fgn2")` object will expand into a Gaussian of length `order*n`, which are the cumulative sums of the the `order` vector of AR1 processes each of length  $n$ . If `order==3`, with weighted AR1 processes (and with the given precision),  $x$ ,  $xx$  and  $xxx$ , then `model="fgn2"` return the vector  $(x + xx + xxx, xx + xxx, xxx)$  where  $\phi_x < \phi_{xx} < \phi_{xxx}$ .

The PC-prior for  $H$  take two arguments  $(U, \alpha)$  where  $\text{Prob}(U < H < 1) = \alpha$ .